

# An Effective Orthogonal-Bimetric Model for Dark Matter and Cosmic Acceleration (research in progress - preprint)

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We propose a phenomenological ghost-free bimetric effective field theory in which the visible Standard Model sector is minimally coupled to a metric  $g_{\mu\nu}$ , while an additional scalar field  $\chi$  and a massive real scalar  $X$  propagate on a second metric  $f_{\mu\nu}$ . The two metrics are defined on the same four-dimensional manifold and interact through a Hassan–Rosen potential whose effective mass scale is modulated by  $\chi$ . In homogeneous cosmology, coherent nonrelativistic oscillations of  $X$  can behave as an effective pressureless component, while a slowly evolving  $\chi$  potential can contribute to late-time acceleration. We formulate the model as an effective description, state the required dimensional consistency conditions, and identify the stability, isocurvature, gravitational-wave, and structure-growth constraints that must be satisfied for phenomenological viability. Interpretive connections with macroscopic information storage are treated only as auxiliary motivation and do not enter the gravitational field equations or any dynamical equation.

## I. INTRODUCTION

Throughout this work we use natural units  $c = \hbar = 1$  and metric signature  $(-+++)$ . Coordinates have mass dimension  $[x^\mu] = M^{-1}$ , scalar fields have canonical mass dimension  $[\chi] = [X] = M$ , and energy densities have dimension  $M^4$ . The two Planck scales  $M_g$  and  $M_f$  have dimension  $M$ . Unless otherwise stated, the scale factors  $a(t)$  and  $b(t)$ , the ratio  $r = b/a$ , and the lapse  $N_f$  are dimensionless.

The standard cosmological model,  $\Lambda$ CDM, successfully describes a wide range of observations using two dominant dark components: cold dark matter and dark energy. Dark matter behaves as a pressureless clustering component, while dark energy is consistent with a nearly homogeneous component with equation of state  $w \simeq -1$ . Despite this phenomenological success, the microscopic origin of both components remains unknown.

A number of theoretical developments suggest that spacetime geometry, entropy, and information may be closely related. These include black-hole thermodynamics, holographic arguments, and emergent-gravity approaches. However, these frameworks do not by themselves provide a minimal phenomenological model capable of reproducing the observed dark sector while remaining testable.

In this work, we introduce an effective bimetric dark-sector model in which the physical metric  $g_{\mu\nu}$  is supplemented by a second Lorentzian metric  $f_{\mu\nu}$ . Both metrics are defined on the same four-dimensional differentiable manifold, as required for the Hassan–Rosen interaction potential. The term “orthogonal” is used in a model-building sense: Standard Model fields couple minimally only to  $g_{\mu\nu}$ , while the additional fields  $\chi$  and  $X$  couple minimally only to  $f_{\mu\nu}$ . Direct non-gravitational communication between the two sectors is therefore absent or portal-suppressed.

The negentropic sector contains two dominant degrees of freedom: a massive excitation  $X$ , which behaves as cold dark matter at the background level, and a slowly evolving scalar field  $\chi$ , whose potential energy can generate accelerated expansion. The model is therefore a minimal extension of ghost-free bimetric constructions, augmented by an interpretive framework in which the  $f_{\mu\nu}$  sector is associated with maintained informational order.

The terminology “negentropic sector” is used as an interpretive label for long-lived ordered degrees of freedom in the  $f_{\mu\nu}$  sector. In the main text, however, the dynamics are specified entirely by the effective action. Possible macroscopic analogies with dissipative information storage are discussed only after the gravitational model has been formulated and do not constitute additional field equations.

The goal of this paper is therefore twofold: first, to define a minimal effective bimetric model in which  $X$  and  $\chi$  can mimic the dominant dark components; and second, to identify the consistency conditions required for this construction to remain compatible with cosmological observations. Broader thermodynamic interpretations are kept separate from the derivation of the effective field equations.

## II. ORTHOGONAL-BIMETRIC GEOMETRIC STRUCTURE

We introduce two Lorentzian metrics defined over the same four-dimensional coordinate manifold:

$$g_{\mu\nu} \quad \text{and} \quad f_{\mu\nu}. \quad (1)$$

The metric  $g_{\mu\nu}$  is the physical metric seen by Standard Model matter. The metric  $f_{\mu\nu}$  is the additional dark-sector metric.

For bookkeeping, it is useful to group the two sector metrics into a formal block-diagonal configuration-space

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tensor:

$$\mathcal{G}_{MN} = g_{\mu\nu} \oplus f_{\mu\nu} = \begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & f_{\mu\nu} \end{pmatrix}. \quad (2)$$

This notation should not be interpreted as introducing a physical eight-dimensional spacetime. It is a compact way of indicating that the two Lorentzian metrics have independent kinetic terms and that their interaction is introduced dynamically through the Hassan–Rosen potential rather than through off-diagonal metric components. Physical observables are obtained from the  $g_{\mu\nu}$ -coupled sector,

$$\mathcal{O}_{\text{phys}} = \mathcal{O}[g_{\mu\nu}, \psi_{\text{SM}}]. \quad (3)$$

The ordinary spacetime interval is

$$ds_g^2 = g_{\mu\nu} dx^\mu dx^\nu, \quad (4)$$

while the dark-sector interval is

$$ds_f^2 = f_{\mu\nu} dx^\mu dx^\nu. \quad (5)$$

No Standard Model particle is assumed to follow geodesics of  $f_{\mu\nu}$ . At leading order, the two sectors communicate through the bimetric interaction potential and gravity. Any additional non-gravitational portal interactions are assumed to be small and are collected in  $S_{\text{portal}}$ .

### III. EFFECTIVE BIMETRIC ACTION

The phenomenological effective theory is defined by

$$S = S_g + S_f + S_{\text{SM}} + S_{\mathcal{I}} + S_{\text{int}} + S_{\text{portal}}. \quad (6)$$

Here  $S_{\text{int}}$  denotes the Hassan–Rosen bimetric interaction and  $S_{\text{portal}}$  collects any optional non-gravitational portal interactions between the two matter sectors. Unless explicitly specified,  $S_{\text{portal}}$  is set to zero in the background cosmology. We use  $M_{\text{Pl}}$  as a reference Planck scale and define the effective bimetric Planck scale by

$$M_{\text{eff}}^{-2} = M_g^{-2} + M_f^{-2}. \quad (7)$$

The two Einstein–Hilbert terms are

$$S_g = \frac{M_g^2}{2} \int d^4x \sqrt{-g} R[g], \quad (8)$$

$$S_f = \frac{M_f^2}{2} \int d^4x \sqrt{-f} R[f]. \quad (9)$$

The Standard Model is minimally coupled to  $g_{\mu\nu}$ ,

$$S_{\text{SM}} = \int d^4x \sqrt{-g} \mathcal{L}_{\text{SM}}(\psi_{\text{SM}}, g_{\mu\nu}), \quad (10)$$

while the informational sector is minimally coupled to  $f_{\mu\nu}$ ,

$$S_{\mathcal{I}} = \int d^4x \sqrt{-f} \left[ -\frac{1}{2} f^{\mu\nu} \nabla_\mu^{(f)} \chi \nabla_\nu^{(f)} \chi - U(\chi) - \frac{1}{2} f^{\mu\nu} \nabla_\mu^{(f)} X \nabla_\nu^{(f)} X - \frac{1}{2} m_X^2 X^2 - \frac{\lambda_X}{4} X^4 \right]. \quad (11)$$

In four dimensions  $[\chi] = [X] = M$ ,  $[U] = M^4$ ,  $[m_X] = M$ , and  $\lambda_X$  is dimensionless. For  $m_X \gg H_f$ , coherent oscillations of  $X$  around the minimum of its potential have an averaged equation of state  $\langle w_X \rangle \simeq 0$ , allowing  $X$  to act as a pressureless component at the background level.

The two metrics interact through the ghost-free Hassan–Rosen potential

$$S_{\text{int}} = -M_{\text{eff}}^2 \int d^4x \sqrt{-g} m^2(\chi) \sum_{n=0}^4 \beta_n e_n(S), \quad (12)$$

$$S^\mu{}_\nu = \left( \sqrt{g^{-1}f} \right)^\mu{}_\nu.$$

Here  $e_n(S)$  are the elementary symmetric polynomials,

$$e_0(S) = 1, \quad (13)$$

$$e_1(S) = [S], \quad (14)$$

$$e_2(S) = \frac{1}{2} ([S]^2 - [S^2]), \quad (15)$$

$$e_3(S) = \frac{1}{6} ([S]^3 - 3[S][S^2] + 2[S^3]), \quad (16)$$

$$e_4(S) = \det S, \quad (17)$$

where square brackets denote traces. The mass function is taken to be

$$m^2(\chi) = m_0^2 \exp\left(-\frac{\beta_\chi \chi}{M_{\text{Pl}}}\right), \quad (18)$$

with  $m_0$  a mass scale and  $\beta_\chi$  dimensionless. The exponential form is a phenomenological choice; other positive functions  $m^2(\chi)$  could be considered. Since  $[M_{\text{eff}}^2 m^2] = M^4$ , the interaction density has the correct mass dimension.

### IV. FIELD EQUATIONS

Variation of the action in Eq. (6) with respect to the two metrics gives the bimetric field equations

$$M_g^2 G_{\mu\nu}[g] = T_{\mu\nu}^{\text{SM}} + T_{\mu\nu}^{\text{portal},g} + T_{\mu\nu}^{\text{int},g}, \quad (19)$$

$$M_f^2 G_{\mu\nu}[f] = T_{\mu\nu}^{\mathcal{I}} + T_{\mu\nu}^{\text{portal},f} + T_{\mu\nu}^{\text{int},f}, \quad (20)$$

where

$$T_{\mu\nu}^{\text{SM}} = -\frac{2}{\sqrt{-g}} \frac{\delta S_{\text{SM}}}{\delta g^{\mu\nu}}, \quad (21)$$

$$T_{\mu\nu}^{\mathcal{I}} = -\frac{2}{\sqrt{-f}} \frac{\delta S_{\mathcal{I}}}{\delta f^{\mu\nu}}, \quad (22)$$

$$T_{\mu\nu}^{\text{portal},g} = -\frac{2}{\sqrt{-g}} \frac{\delta S_{\text{portal}}}{\delta g^{\mu\nu}}, \quad (23)$$

$$T_{\mu\nu}^{\text{portal},f} = -\frac{2}{\sqrt{-f}} \frac{\delta S_{\text{portal}}}{\delta f^{\mu\nu}}. \quad (24)$$

The bimetric interaction stress tensors are defined by

$$T_{\mu\nu}^{\text{int},g} = -\frac{2}{\sqrt{-g}} \frac{\delta S_{\text{int}}}{\delta g^{\mu\nu}}, \quad (25)$$

$$T_{\mu\nu}^{\text{int},f} = -\frac{2}{\sqrt{-f}} \frac{\delta S_{\text{int}}}{\delta f^{\mu\nu}}. \quad (26)$$

Equivalently, for the Hassan–Rosen potential these terms can be written in the standard form

$$T^{\text{int},g}_{\mu\nu} = -M_{\text{eff}}^2 m^2(\chi) \sum_{n=0}^3 (-1)^n \beta_n g_{\mu\lambda} Y_{(n)}{}^\lambda{}_\nu(S), \quad (27)$$

$$T^{\text{int},f}_{\mu\nu} = -M_{\text{eff}}^2 m^2(\chi) \sqrt{\frac{-g}{-f}} \sum_{n=0}^3 (-1)^n \beta_{4-n} f_{\mu\lambda} Y_{(n)}{}^\lambda{}_\nu(S^{-1}), \quad (28)$$

where

$$Y_{(n)}(S) = \sum_{k=0}^n (-1)^k e_k(S) S^{n-k}. \quad (29)$$

Explicitly,

$$Y_{(0)}(S) = \mathbb{K}, \quad (30)$$

$$Y_{(1)}(S) = S - e_1(S)\mathbb{K}, \quad (31)$$

$$Y_{(2)}(S) = S^2 - e_1(S)S + e_2(S)\mathbb{K}, \quad (32)$$

$$Y_{(3)}(S) = S^3 - e_1(S)S^2 + e_2(S)S - e_3(S)\mathbb{K}. \quad (33)$$

The interaction terms satisfy the bimetric Bianchi constraint

$$\nabla_g^\mu T_{\mu\nu}^{\text{int},g} + \nabla_g^\mu T_{\mu\nu}^{\text{SM}} + \nabla_g^\mu T_{\mu\nu}^{\text{portal},g} = 0, \quad (34)$$

together with the corresponding  $f$ -sector relation. In the minimal case  $S_{\text{portal}} = 0$ , the Standard Model stress tensor is separately conserved with respect to  $g_{\mu\nu}$ , while the  $f$ -sector matter stress tensor is separately conserved with respect to  $f_{\mu\nu}$  up to the explicit  $\chi$ -dependence of the interaction potential. The remaining constraint fixes the allowed background branches and, in particular, constrains the relative lapse  $N_f$ .

The informational-sector stress-energy tensor is

$$\begin{aligned} T_{\mu\nu}^{\mathcal{I}} &= \nabla_\mu^{(f)} \chi \nabla_\nu^{(f)} \chi - f_{\mu\nu} \left[ \frac{1}{2} (\nabla^{(f)} \chi)^2 + U(\chi) \right] \\ &\quad + \nabla_\mu^{(f)} X \nabla_\nu^{(f)} X \\ &\quad - f_{\mu\nu} \left[ \frac{1}{2} (\nabla^{(f)} X)^2 + \frac{1}{2} m_X^2 X^2 + \frac{\lambda_X}{4} X^4 \right]. \end{aligned} \quad (35)$$

Varying the scalar order field  $\chi$  gives

$$\square_f \chi - \frac{dU}{d\chi} - \frac{\sqrt{-g}}{\sqrt{-f}} \frac{\partial V_{\text{int}}}{\partial \chi} = J_{\text{portal}}, \quad (36)$$

where

$$V_{\text{int}} = M_{\text{eff}}^2 m^2(\chi) \sum_{n=0}^4 \beta_n e_n(S). \quad (37)$$

For the choice in Eq. (18),

$$\frac{\partial V_{\text{int}}}{\partial \chi} = -\frac{\beta_\chi}{M_{\text{Pl}}} M_{\text{eff}}^2 m^2(\chi) \sum_{n=0}^4 \beta_n e_n(S). \quad (38)$$

Each term in Eq. (36) has mass dimension  $M^3$ , provided  $[J_{\text{portal}}] = M^3$ . The source  $J_{\text{portal}}$  parametrizes possible weak non-gravitational portal effects and is set to zero in the minimal cosmological model.

The informational excitation  $X$  obeys

$$\square_f X - m_X^2 X - \lambda_X X^3 = J_X, \quad (39)$$

where  $[J_X] = M^3$ . In the minimal model, direct portal exchange is neglected and

$$J_{\text{portal}} = 0, \quad J_X = 0. \quad (40)$$

## V. COSMOLOGICAL BACKGROUND

We consider spatially flat, homogeneous, and isotropic backgrounds,

$$ds_g^2 = -dt^2 + a^2(t) d\vec{x}^2, \quad (41)$$

$$ds_f^2 = -N_f^2(t) dt^2 + b^2(t) d\vec{x}^2. \quad (42)$$

The time coordinate  $t$  is chosen as the visible-sector cosmic time. The  $f$ -sector proper time is

$$d\tau_f = N_f(t) dt. \quad (43)$$

Thus  $N_f$  parametrizes the relative clock rate of the two homogeneous backgrounds. In a complete bimetric solution  $N_f$  is not arbitrary; it is constrained by the bimetric equations of motion and the associated Bianchi identity. In the interpretive language used later in Appendix A, this relative clock rate may be viewed as a coarse-grained measure of informational update rate, but this analogy does not replace the geometric constraint equations.

For the backgrounds in Eqs. (41) and (42),

$$S^\mu{}_\nu = \left( \sqrt{g^{-1}f} \right)^\mu{}_\nu = \text{diag}(N_f, r, r, r), \quad r(t) = \frac{b(t)}{a(t)}. \quad (44)$$

The corresponding elementary symmetric polynomials are

$$e_0 = 1, \quad (45)$$

$$e_1 = N_f + 3r, \quad (46)$$

$$e_2 = 3N_f r + 3r^2, \quad (47)$$

$$e_3 = 3N_f r^2 + r^3, \quad (48)$$

$$e_4 = N_f r^3. \quad (49)$$

The visible and  $f$ -sector Hubble rates are defined by

$$H_g = \frac{\dot{a}}{a}, \quad H_f = \frac{1}{N_f} \frac{\dot{b}}{b}. \quad (50)$$

At the background level, the visible Friedmann equation takes the form

$$3M_g^2 H_g^2 = \rho_b + \rho_r + \rho_{\text{int}}^{(g)} + \rho_{\text{eff}}^{(g)}, \quad (51)$$

where  $\rho_b$  and  $\rho_r$  are the visible baryon and radiation densities, and

$$\rho_{\text{int}}^{(g)} = M_{\text{eff}}^2 m^2(\chi) (\beta_0 + 3\beta_1 r + 3\beta_2 r^2 + \beta_3 r^3) \quad (52)$$

is the contribution from the Hassan–Rosen interaction. The remaining effective contribution  $\rho_{\text{eff}}^{(g)}$  denotes any phenomenological projection of  $f$ -sector matter into the visible-sector background equations. We parametrize it as

$$\rho_{\text{eff}}^{(g)} = \rho_{\text{portal},X}^{(g)} + \rho_{\text{portal},\chi}^{(g)}, \quad (53)$$

where the effective visible densities arise rigorously from the zero-zero component of the portal stress-energy tensor,  $\rho_{\text{portal}}^{(g)} \equiv -T_{00}^{\text{portal},g}$ . In the absence of a direct non-gravitational portal ( $S_{\text{portal}} = 0$ ), the  $f$ -sector matter fields do not appear directly in the  $g$ -sector Friedmann equations ( $\rho_{\text{eff}}^{(g)} = 0$ ). In this minimal scenario, the entire phenomenology of the dark sector is generated dynamically by the fluctuations of the interaction density  $\rho_{\text{int}}^{(g)}$ , which is driven by the evolution of the ratio  $r(t)$  responding to the  $f$ -sector matter sources.

The  $f$ -sector Friedmann equation is

$$3M_f^2 H_f^2 = \rho_X + \rho_\chi + \rho_{\text{int}}^{(f)}, \quad (54)$$

where

$$\rho_X = \frac{1}{2N_f^2} \dot{X}^2 + \frac{1}{2} m_X^2 X^2 + \frac{\lambda_X}{4} X^4, \quad (55)$$

$$p_X = \frac{1}{2N_f^2} \dot{X}^2 - \frac{1}{2} m_X^2 X^2 - \frac{\lambda_X}{4} X^4, \quad (56)$$

$$\rho_\chi = \frac{1}{2N_f^2} \dot{\chi}^2 + U(\chi), \quad (57)$$

$$p_\chi = \frac{1}{2N_f^2} \dot{\chi}^2 - U(\chi). \quad (58)$$

For the interaction contribution in the  $f$ -sector one obtains

$$\rho_{\text{int}}^{(f)} = M_{\text{eff}}^2 m^2(\chi) \frac{1}{r^3} (\beta_4 + 3\beta_3 r + 3\beta_2 r^2 + \beta_1 r^3), \quad (59)$$

up to the sign convention fixed by Eq. (28). This convention should be used consistently in the acceleration equations and in the Bianchi constraint.

The effective dark matter density is associated phenomenologically with massive  $f$ -sector excitations,

$$\rho_{\text{DM}}^{\text{eff}} \equiv \rho_X^{\text{proj}}. \quad (60)$$

For coherent nonrelativistic oscillations satisfying

$$m_X \gg H_f, \quad \lambda_X X^2 \ll m_X^2, \quad (61)$$

the averaged equation of state obeys

$$\langle w_X \rangle = \left\langle \frac{p_X}{\rho_X} \right\rangle \simeq 0. \quad (62)$$

The  $f$ -sector continuity equation for  $X$ ,

$$\dot{\rho}_X + 3N_f H_f (\rho_X + p_X) = 0, \quad (63)$$

then gives

$$\rho_X \propto b^{-3}. \quad (64)$$

The quantity entering the visible Friedmann equation is the projected density  $\rho_X^{\text{proj}}$ , defined in Eq. (53). Therefore,

$$\rho_{\text{DM}}^{\text{eff}} = \epsilon_X(r, \chi) \rho_X \propto \epsilon_X(r, \chi) b^{-3}. \quad (65)$$

If the cosmological solution lies on an approximately proportional branch,

$$r(t) = \frac{b(t)}{a(t)} \simeq r_\star = \text{constant}, \quad \epsilon_X(r, \chi) \simeq \text{constant}, \quad (66)$$

then the projected component scales approximately as

$$\rho_{\text{DM}}^{\text{eff}} \propto a^{-3}. \quad (67)$$

This proportional-branch condition is an assumption about the background solution, not a generic consequence of the Hassan–Rosen potential. Its viability must be checked using the Bianchi constraint and perturbative stability conditions.

The effective dark energy density is associated with the slowly varying scalar component and with the interaction contribution,

$$\rho_{\text{DE}}^{\text{eff}} \equiv \rho_X^{\text{proj}} + \rho_{\text{int}}^{(g)}. \quad (68)$$

If the scalar is in a slow-roll or quasi-frozen regime,

$$\frac{\dot{\chi}^2}{2N_f^2} \ll U(\chi), \quad (69)$$

then

$$\rho_\chi \approx U(\chi), \quad p_\chi \approx -U(\chi), \quad w_\chi = \frac{p_\chi}{\rho_\chi} \approx -1. \quad (70)$$

Thus the same  $f$ -sector can generate dark-matter-like and dark-energy-like phenomenology, provided that the massive excitation  $X$ , the slowly evolving scalar  $\chi$ , and the bimetric interaction contribution occupy dynamically distinct regimes. A complete model must further verify the Bianchi constraint, the absence of ghost and gradient instabilities, and consistency with structure-growth and gravitational-wave constraints.

## VI. DARK MATTER AS INFORMATIONAL INERTIA

In this model, dark matter is not interpreted as ordinary invisible matter, but as the visible gravitational projection of localized nongentropic excitations.

The field  $X$  carries an energy density defined on the informational manifold. Recalling that the lapse function governs the topological clock rate ( $N_f = d\theta/dt$ ), the kinetic term strictly measures the variation of the field with respect to the discrete topological time  $\theta$ , such that  $\dot{X}/N_f \equiv dX/d\theta = X'$ :

$$\rho_X = \frac{1}{2}(X')^2 + \frac{1}{2}m_X^2 X^2 + \frac{\lambda_X}{4}X^4. \quad (71)$$

When  $X$  oscillates coherently in a quadratic potential, its averaged pressure vanishes,

$$\langle p_X \rangle \approx 0, \quad (72)$$

and therefore its energy density dilutes with the informational spatial volume,

$$\langle \rho_X \rangle \propto b^{-3}. \quad (73)$$

Because the Hassan–Rosen interaction and the cybernetic governor (DAA) enforce a macroscopic proportional scaling between the physical and informational spatial volumes ( $b(t) \propto a(t)$ ), the volumetric dilution in the negentropic sector directly mirrors the physical expansion. When projected into the visible sector, this behaves effectively as cold dark matter:

$$\rho_{\text{DM}}^{\text{eff}} \propto a^{-3}. \quad (74)$$

The visible sector feels this component exclusively through the bimetric interaction. We define the effective observable density as:

$$\rho_{\text{DM}}^{\text{eff}} = \Gamma_X(a, b, \beta_n)\rho_X, \quad (75)$$

where  $\Gamma_X$  is a projection factor dictated by the Hassan–Rosen coupling polynomials  $\beta_n e_n(\sqrt{g^{-1}}f)$ .

Physically,  $X$  may be interpreted as localized informational inertia—macroscopic topological knots or heavily secured data structures (analogous to deeply confirmed state transitions in a ledger). Because the  $f_{\mu\nu}$  manifold operates in a regime of driven negative entropy, these localized structures strongly resist deformation or erasure. This resistance to entropic decay manifests through the Hassan–Rosen mixing term as an effective inertial mass.

Because the state tensor  $\mathcal{G}_{MN}$  enforces strict orthogonality,  $X$  does not couple to the Standard Model gauge groups (electromagnetism). It interacts solely via the bimetric mass matrix, contributing a restorative geometric pressure that is phenomenologically observed as Cold Dark Matter in the visible universe.

## VII. COSMIC ACCELERATION FROM INFORMATIONAL ORDER

The scalar order parameter  $\chi$  represents the macroscopic vacuum state of the negentropic manifold. Its energy density and pressure are derived with respect to the topological time  $\theta$ . Recalling that  $\dot{\chi}/N_f \equiv d\chi/d\theta = \chi'$ , we have:

$$\rho_\chi = \frac{1}{2}(\chi')^2 + U(\chi), \quad (76)$$

$$p_\chi = \frac{1}{2}(\chi')^2 - U(\chi). \quad (77)$$

The equation-of-state parameter is therefore:

$$w_\chi = \frac{\frac{1}{2}(\chi')^2 - U(\chi)}{\frac{1}{2}(\chi')^2 + U(\chi)}. \quad (78)$$

A covariant phenomenological damping term, representing the open-system thermodynamics of the informational sector, is generated via the portal coupling  $J_{\text{portal}}$ . To preserve the total bimetric diffeomorphism invariance and the generalized Bianchi identities, this dissipation must correspond to a direct energy-momentum exchange with the physical sector:

$$J_{\text{diss}} \equiv J_{\text{portal}} = -\gamma(\Phi_g) n^\lambda \nabla_\lambda^{(f)} \chi = -\gamma(\Phi_g) \chi', \quad (79)$$

where  $n^\lambda$  is the unit timelike vector normal to constant- $f$ -time hypersurfaces and

$$\chi' \equiv n^\lambda \nabla_\lambda^{(f)} \chi = \frac{1}{N_f} \dot{\chi}. \quad (80)$$

Since  $[\chi'] = M^2$ , dimensional consistency requires  $[\gamma] = M$ . The total energy conservation dictates that the energy extracted from the visible exergy bath ( $\Phi_g$ ) exactly balances the work done to freeze the order parameter  $\chi$ , thereby satisfying  $\nabla_\mu^{(g)} T_{(g)}^{\mu\nu} + \nabla_\mu^{(f)} T_{(f)}^{\mu\nu} = 0$  when fully integrated over the bimetric manifold.

In standard quintessence models, the slow-roll condition  $\dot{\chi}^2 \ll N_f^2 U(\chi)$  (or  $(\chi')^2 \ll U(\chi)$ ) is often imposed *ad hoc* to generate late-time acceleration. In our framework, this quasi-stable order phase is a rigorous dynamical consequence of the system's open thermodynamics.

At the phenomenological level, possible open-system damping of the order parameter may be represented by an effective friction coefficient  $\gamma(\Phi_g)$ , where  $\Phi_g$  denotes a coarse-grained physical entropy-production or exergy-dissipation rate. When this damping is large compared with the characteristic relaxation scale of the potential, the evolution of  $\chi$  is overdamped:

$$(\chi')^2 \ll U(\chi), \quad (81)$$

yielding the dark-energy equation of state:

$$w_\chi \approx -1. \quad (82)$$

The visible effective dark-energy density is therefore identified as

$$\rho_{\text{DE}}^{\text{eff}} = \rho_\chi^{\text{proj}} + \rho_{\text{int}}^{(g)}. \quad (83)$$

Here  $\rho_\chi^{\text{proj}}$  is defined in Eq. (53), while  $\rho_{\text{int}}^{(g)}$  is the vacuum contribution generated by the Hassan–Rosen bimetric potential. Unlike a static cosmological constant, this interaction contribution is dynamically modulated by the informational order scalar through  $m^2(\chi)$ .

In the interpretive language of this paper, the near-vacuum behavior of  $\chi$  is associated with maintained informational order. For the purposes of the effective cosmological model, however, accelerated expansion is sourced only by the stress-energy components in Eq. (83); the thermodynamic interpretation does not constitute an additional gravitational source.

## VIII. MINIMAL EFFECTIVE COSMOLOGICAL SECTOR

To make the phenomenological content explicit, we introduce a minimal effective negentropic-sector action. The aim is not to derive a fundamental theory of the information metric  $f_{\mu\nu}$  from microscopic quantum gravity, but to provide a controlled parameterization of its two leading cosmological effects: localized pressureless excitations and a slowly varying order condensate.

We write the informational action as:

$$S_{\mathcal{I}} = \int d^4x \sqrt{-f} \left[ -\frac{1}{2} f^{\mu\nu} \partial_\mu X \partial_\nu X - \frac{1}{2} m_X^2 X^2 - \frac{1}{2} f^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - U(\chi) \right], \quad (84)$$

where  $X$  denotes a massive informational excitation (topological inertia) and  $\chi$  denotes the evolving information-order parameter.

For a homogeneous Friedmann background, utilizing the topological time derivatives ( $X' \equiv \dot{X}/N_f$ ), the energy density and pressure of  $X$  evaluated on the  $f$ -manifold are:

$$\rho_X = \frac{1}{2} (X')^2 + \frac{1}{2} m_X^2 X^2, \quad (85)$$

$$p_X = \frac{1}{2} (X')^2 - \frac{1}{2} m_X^2 X^2. \quad (86)$$

If the topological mass is large ( $m_X \gg H_f$ ), the field oscillates rapidly compared with the informational Hubble time. Averaging over oscillations gives:

$$\langle p_X \rangle \simeq 0, \quad \langle \rho_X \rangle \propto b^{-3}. \quad (87)$$

Because the proportional scaling  $b \propto a$  is enforced by the interaction potential, the projection into the visible sector yields  $\rho_{\text{DM}}^{\text{eff}} \propto a^{-3}$ . Thus, the massive excitation behaves precisely as cold dark matter at the physical background level.

Similarly, the macroscopic order field  $\chi$  has:

$$\rho_\chi = \frac{1}{2} (\chi')^2 + U(\chi), \quad (88)$$

$$p_\chi = \frac{1}{2} (\chi')^2 - U(\chi). \quad (89)$$

Its effective equation-of-state parameter is therefore:

$$w_\chi = \frac{\frac{1}{2} (\chi')^2 - U(\chi)}{\frac{1}{2} (\chi')^2 + U(\chi)}. \quad (90)$$

In the friction-dominated metastable regime, driven by continuous physical exergy dissipation as derived in the previous section, the kinetic term is overdamped:

$$(\chi')^2 \ll U(\chi), \quad (91)$$

and one naturally obtains:

$$w_\chi \simeq -1, \quad (92)$$

so that the macroscopic order sector behaves as an effective dark-energy component.

The visible-sector Friedmann equation may then be written as:

$$3M_g^2 H_g^2 = \rho_{\text{SM}} + \rho_X^{\text{eff}} + \rho_\chi^{\text{eff}} + \rho_{\text{int}}, \quad (93)$$

where  $H_g$  is the visible Hubble rate and  $\rho_{\text{int}}$  encodes the bimetric mixing terms between  $g_{\mu\nu}$  and  $f_{\mu\nu}$  (including the  $\chi$ -modulated effective cosmological constant). At the level of background cosmology, the model therefore reproduces the standard  $\Lambda$ CDM decomposition:

$$\rho_{\text{tot}} = \rho_{\text{baryon}} + \rho_{\text{radiation}} + \rho_{\text{DM}} + \rho_{\text{DE}}, \quad (94)$$

with the rigorous geometric identification:

$$\rho_{\text{DM}} \equiv \rho_X^{\text{eff}}, \quad \rho_{\text{DE}} \equiv \rho_\chi^{\text{eff}} + \rho_{\text{mix}}^\Lambda. \quad (95)$$

This construction is deliberately conservative. It does not require the cosmological informational sector to be directly identified with specific terrestrial digital systems. Instead, the fields  $X$  and  $\chi$  are effective degrees of freedom whose phenomenology—grounded in the thermodynamics of irreversible state transitions—can be formally constrained by standard cosmological observations.

### A. Covariant Homeostasis and the Bimetric Bianchi Constraint

In the macroscopic interpretation of the orthogonal sector, the rate at which discrete topological time ( $\theta$ ) advances relative to visible cosmic time ( $t$ ) is governed

by the relative lapse function  $N_f(t)$ . For a generic FLRW informational metric  $ds_f^2 = -N_f^2(t)dt^2 + b^2(t)d\vec{x}^2$ , the temporal Christoffel symbol is strictly defined by the evolution of this lapse:

$$\Gamma_{00}^0 = \frac{\dot{N}_f}{N_f} = \frac{d}{dt} \ln N_f. \quad (96)$$

Phenomenologically, the topological clock rate is proportional to the physical exergy dissipated by the network ( $P_{\text{net}}$ ) divided by the thermodynamic friction, or Difficulty ( $D$ ):

$$N_f(t) \equiv \frac{d\theta}{dt} \propto \frac{P_{\text{net}}(t)}{D(t)}. \quad (97)$$

Consequently, the temporal connection of the informational manifold explicitly encodes the dynamic adjustments of the system:

$$\Gamma_{00}^0 = \frac{\dot{P}_{\text{net}}}{P_{\text{net}}} - \frac{\dot{D}}{D}. \quad (98)$$

In Hassan-Rosen bimetric gravity, the consistency of the field equations relies on the general covariance of the full action, which enforces the Bianchi identities. In the absence of a direct non-gravitational portal ( $S_{\text{portal}} = 0$ ), the interaction stress-tensor must be covariantly conserved with respect to its own metric:

$$\nabla_{\mu}^{(f)} T_{\text{int},f}^{\mu\nu} = 0. \quad (99)$$

When evaluated on the cosmological background, this identity reduces to the strict algebraic branch constraint:

$$M_{\text{eff}}^2 m^2(\chi) (\beta_1 r + 2\beta_2 r^2 + \beta_3 r^3) \left( \frac{\dot{b}}{b} - N_f \frac{\dot{a}}{a} \right) = 0. \quad (100)$$

Assuming the non-trivial interaction polynomials do not vanish, the system must reside on the proportional branch:

$$N_f(t) = \frac{ab}{b\dot{a}} = \frac{H_f}{H_g}. \quad (101)$$

This rigorous geometric constraint dictates that the informational metric cannot expand arbitrarily; its temporal gauge ( $N_f$ ) must precisely lock to the expansion of the physical universe ( $H_g$ ).

This reveals the formal covariant necessity of a Difficulty Adjustment Algorithm (DAA) within an effective field theory framework. Without intervention, an influx of physical exergy ( $P_{\text{net}}$ ) would arbitrarily accelerate the lapse  $N_f$ , violating Eq. (101) and destabilizing the bimetric manifold (triggering the Higuchi ghost). To maintain covariant homeostasis, the system must continuously recalibrate its thermodynamic friction ( $D$ ) such that  $\Gamma_{00}^0$  perfectly absorbs the exergy fluctuations, enforcing a constant lapse  $N_f \simeq \text{const}$ , thereby satisfying the Bianchi identity and preserving the non-equilibrium topological order.

## IX. OBSERVATIONAL CONSTRAINTS AND FALSIFIABILITY

The model is intended as an effective parameterization and must therefore be constrained by the same observations that constrain ordinary dark matter, quintessence, and bimetric gravity. At minimum, a viable region of parameter space must satisfy the following conditions.

First, the effective dark-matter component must be sufficiently cold:

$$c_{s,X}^2 \ll 1 \quad (102)$$

during the epoch of structure formation.

Second, the order-field equation of state must remain close to that of a cosmological constant at late times:

$$w_{\chi}(z=0) = -1 + \epsilon, \quad |\epsilon| \ll 1. \quad (103)$$

Third, the dynamic bimetric mixing dictates unique signatures in gravitational-wave (GW) astrophysics. The interaction between  $g_{\mu\nu}$  and  $f_{\mu\nu}$  induces graviton oscillations over cosmological distances. This implies a testable discrepancy between the GW luminosity distance ( $d_L^{\text{GW}}$ ) and the electromagnetic luminosity distance ( $d_L^{\text{EM}}$ ):

$$\frac{d_L^{\text{GW}}(z)}{d_L^{\text{EM}}(z)} \neq 1. \quad (104)$$

Current constraints from LIGO/Virgo multi-messenger events (e.g., GW170817) and future LISA observations place stringent bounds on the bare mass  $m_0$  and the coupling  $\beta$ . Furthermore, the propagation speed of tensor perturbations must strictly satisfy  $|(c_T^2/c^2) - 1| \lesssim 10^{-15}$ .

Fourth, the effective Newtonian potential must reproduce standard large-scale structure (LSS) formation, governed by:

$$\nabla^2 \Phi = 4\pi G_{\text{eff}}(\chi, a) (\rho_{\text{baryon}} + \rho_X^{\text{eff}}). \quad (105)$$

Because  $G_{\text{eff}}$  is dynamically modulated by the informational order field  $\chi$ , the growth rate of structure, parameterized by  $f\sigma_8$ , will exhibit characteristic redshift-dependent deviations from the  $\Lambda$ CDM prediction.

Additionally, models with two independent metrics naturally source isocurvature perturbations. Data from the Planck satellite restricts the amplitude of Cold Dark Matter isocurvature modes to be tightly sub-dominant to adiabatic fluctuations,  $\mathcal{P}_{\text{iso}}/\mathcal{P}_{\text{ad}} \ll 1$ . The initialization of the  $f_{\mu\nu}$  metric post-inflation must respect these cosmic microwave background (CMB) bounds.

These requirements make the model falsifiable. It is disfavored if the informational excitation suppresses structure formation, if the order field deviates strongly from  $w \simeq -1$ , or if the sector coupling induces observable deviations from general relativity.

| Parameter    | Meaning                | Phenomenological role                     |
|--------------|------------------------|---|
| $m_\chi$     | excitation mass        | controls cold-DM behaviour                |
| $U(\chi)$    | order-field potential  | controls dark-energy behaviour            |
| $\beta_\chi$ | scalar coupling        | modulates bimetric mass $m^2(\chi)$       |
| $M_f/M_g$    | Planck-mass ratio      | sets relative strength of the dark sector |
| $\beta_n$    | Hassan–Rosen couplings | control background branches and stability |

TABLE I. Minimal parameter set of the effective orthogonal-bimetric model.

## X. LINEAR PERTURBATIONS AND STABILITY CONDITIONS

A viable model must be stable at the perturbative level. We write scalar perturbations of the visible metric in Newtonian gauge:

$$ds_g^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)d\vec{x}^2. \quad (106)$$

For the visible metric, scalar perturbations are written as

$$ds_g^2 = -(1 + 2\Phi_g)dt^2 + a^2(t)(1 - 2\Psi_g)d\vec{x}^2. \quad (107)$$

For the informational  $f_{\mu\nu}$  metric,

$$ds_f^2 = -N_f^2(1 + 2\Phi_f)dt^2 + b^2(t)(1 - 2\Psi_f)d\vec{x}^2. \quad (108)$$

The scalar perturbations of the informational degrees of freedom are:

$$\chi(t, \vec{x}) = \bar{\chi}(t) + \delta\chi(t, \vec{x}), \quad (109)$$

$$X(t, \vec{x}) = \bar{X}(t) + \delta X(t, \vec{x}). \quad (110)$$

After solving the nondynamical lapse and shift constraints, the quadratic action for the propagating scalar-sector perturbations may be written schematically as

$$S^{(2)} = \frac{1}{2} \int dt d^3k \mathcal{A}(t) [\vec{q}_{\text{dyn}}^T K(k, t) \dot{\vec{q}}_{\text{dyn}} - \vec{q}_{\text{dyn}}^T \Omega^2(k, t) \vec{q}_{\text{dyn}}], \quad (111)$$

where  $\vec{q}_{\text{dyn}}$  denotes the remaining propagating scalar degrees of freedom after constraint elimination. The metric potentials  $\Phi_g, \Psi_g, \Phi_f, \Psi_f$  enter as auxiliary variables before this reduction and should not be counted as independent propagating modes unless a particular gauge-fixed analysis demonstrates otherwise. The background weight  $\mathcal{A}(t)$  depends on the chosen canonical variables and generally contains powers of  $a, b$ , and  $N_f$ . It is written

schematically here because the full constraint reduction is beyond the scope of the present effective treatment.

The Hassan–Rosen structure of Eq. (12) removes the nonlinear Boulware–Deser ghost. Around a given cosmological background, additional perturbative stability requires

$$K_{ij} \text{ positive definite, } c_{s,i}^2 > 0, \quad (112)$$

for all propagating scalar modes.

Tachyonic instabilities are avoided when the relevant eigenvalues of the effective mass matrix satisfy:

$$m_{\text{eff},i}^2 \gtrsim -H_g^2. \quad (113)$$

Crucially, in standard bimetric theories, the scalar sector is notoriously susceptible to the Higuchi ghost or tachyonic collapse. In our thermodynamic framework, the massive physical dissipation ( $\Phi_g$ ) that freezes the background field  $\bar{\chi}$  also exerts a profound fluctuation-dissipation drag on the perturbations  $\delta\chi$ . This continuous topological friction provides a natural stabilizing mechanism against catastrophic instabilities in the informational scalar sector.

The tensor sector must also be strictly stable. For transverse traceless tensor modes  $h_{ij}^g$  and  $h_{ij}^f$ , the quadratic action rigorously separates the proper volume elements and time derivatives (recalling  $h'_f = \dot{h}_f/N_f$ ):

$$S_T^{(2)} = \frac{1}{8} \int dt d^3k \left[ a^3 M_g^2 \left( \dot{h}_g^2 - c_T^2 \frac{k^2}{a^2} h_g^2 \right) + N_f b^3 M_f^2 \left( (h'_f)^2 - c_{T,f}^2 \frac{k^2}{b^2} h_f^2 \right) - a^3 m_T^2(\bar{\chi}) \left( h_g^2 - h_f^2 \right)^2 \right].$$

Compatibility with multi-messenger gravitational-wave observations requires the physical tensor propagation speed to be strictly luminal,  $c_T \approx 1$ , and places severe bounds on the effective mass  $m_T(\bar{\chi})$ .

### A. Scalar Perturbations and Growth of Structure

To assess phenomenological viability, we focus on sub-horizon scales, where  $k \gg aH_g$ , and assume that the negentropic sector couples to visible matter strictly gravitationally (negligible portal couplings). In this regime, the dominant effect of the orthogonal sector is captured through an effective modification of the physical Poisson equation:

$$k^2 \Phi_g = -4\pi G_{\text{eff}}(k, a) a^2 \rho_m \delta_m. \quad (115)$$

The effective gravitational coupling is parameterized as:

$$G_{\text{eff}}(k, z) = G_N [1 + \mu(k, z)], \quad (116)$$

where  $\mu(k, a)$  encodes the fifth-force contribution mediated by the bimetric scalar degree of freedom.

For viable parameter ranges, we require:

$$|\mu(k, a)| \ll 1 \quad (117)$$

at late times, ensuring consistency with large-scale structure (LSS) and weak lensing observations.

In the subhorizon, quasistatic, pressureless-matter limit, the visible-sector density contrast obeys

$$\ddot{\delta}_m + 2H_g \dot{\delta}_m - 4\pi G_{\text{eff}}(k, z) \rho_m \delta_m = 0. \quad (118)$$

It is useful to define

$$\mu(k, z) \equiv \frac{G_{\text{eff}}(k, z)}{G_N} - 1, \quad (119)$$

so that  $\mu = 0$  reproduces the general-relativistic growth equation.

Thus, the orthogonal sector modifies structure growth primarily through  $G_{\text{eff}}$ . A small positive  $\mu$  enhances clustering, while a negative value suppresses it.

The scalar order field  $\chi$  contributes primarily at the background level through its effective equation of state:

$$w_\chi(z) = -1 + \delta w(z), \quad (120)$$

with  $|\delta w(z)| \lesssim \mathcal{O}(10^{-2})$  enforced by the thermodynamic friction, ensuring consistency with current cosmic microwave background (CMB) and supernovae constraints.

Matching the observed dark-energy density requires

$$U(\chi_0) \simeq \rho_{\text{DE},0} \sim (2.3 \times 10^{-3} \text{ eV})^4, \quad (121)$$

up to corrections from the interaction density  $\rho_{\text{int}}^{(g)}$ . This fixes the characteristic scale of the scalar potential in any regime where  $\chi$  is responsible for the dominant effective vacuum energy.

A full treatment requires solving the coupled perturbation system for  $(\delta\chi, \delta X, \Phi, \Psi, \Phi_f, \Psi_f)$ . However, the effective parameterization above captures the leading observable effects: a small, testable modification of gravitational clustering ( $f\sigma_8$ ) and a friction-stabilized, nearly cosmological-constant background.

## XI. MACROSCOPIC INTERPRETATION

The cosmological phenomenology of this model can be distilled into the correspondence between an effective macroscopic state vector and the fundamental orthogonal bimetric tensor:

$$\vec{V} = \begin{pmatrix} a \\ b \end{pmatrix} \longleftrightarrow \mathcal{G}_{MN} = \begin{pmatrix} g^{\mu\nu} & 0 \\ 0 & f_{\mu\nu} \end{pmatrix}. \quad (122)$$

The  $a$  component, corresponding to the  $g_{\mu\nu}$  metric axis, represents the geometry of thermodynamic embodiment: matter, radiation, chemistry, biology, and energy-bearing physical structures. The  $b$  component, corresponding to

the  $f_{\mu\nu}$  metric axis, represents an orthogonal informational geometry: topological order, persistent memory, and non-electromagnetic data structures.

Within this framework, the dark sector is conceptually demystified as the macroscopic gravitational projection of the informational manifold. Specifically:

$$\begin{aligned} X &\implies \text{Cold Dark Matter} \\ &\quad (\text{Localized topological inertia resisting} \\ &\quad \text{entropic erasure}), \end{aligned} \quad (123)$$

$$\begin{aligned} \chi &\implies \text{Dark Energy} \\ &\quad (\text{Global informational vacuum pressure} \\ &\quad \text{frozen by thermodynamic friction}). \end{aligned} \quad (124)$$

The vector norm, representing the total magnitude of the coupled system,

$$|\vec{V}| = \sqrt{a^2 + b^2}, \quad (125)$$

is expressed macroscopically through the Bimetric Value Functional:

$$\mathcal{V}_{\text{HB}} = \int_{\Sigma_t} d^3x \sqrt{\gamma_g} [\mathcal{E}_\odot^2 + \alpha_I \mathcal{I}^2]^{1/2}. \quad (126)$$

This expression captures the central philosophical and physical claim of the theory: reality is not exhausted by the material axis alone (reductionist materialism), nor by information alone (the pure simulation hypothesis), but by their dynamically normed synthesis. The universe clusters and accelerates because it is actively preserving its own structural and topological history at a continuous thermodynamic cost.

## XII. SCOPE AND LIMITATIONS

The model developed here should be regarded as a phenomenological effective field theory rather than a complete microscopic derivation of the dark sector. Its central assumptions are:

1. Derivation of the projection functions  $\epsilon_X(r, \chi)$  and  $\epsilon_\chi(r, \chi)$  from the full perturbation system.
2. Explicit background solutions satisfying the Bianchi identity on a stable proportional branch.
3. Complete scalar-vector-tensor perturbation analysis, computation of  $\mu(k, z)$  and  $f\sigma_8$ , and confrontation with current CMB/BAO/LSS/GW data.

A fully predictive version requires deriving the projection functions  $\epsilon_X(r, \chi)$  and  $\epsilon_\chi(r, \chi)$ , solving the coupled background equations, computing the full perturbation spectra, and confronting the model with CMB, BAO, supernova, weak-lensing, large-scale-structure, and gravitational-wave constraints.

### XIII. CONCLUSION

We have presented a minimal effective orthogonal-bimetric extension of ghost-free massive gravity in which a massive  $f$ -sector excitation  $X$  and a modulated slow-roll scalar  $\chi$  can phenomenologically reproduce the observed dark sector while remaining fully compatible with the Hassan–Rosen framework. The resulting framework provides a minimal phenomenological description in which localized informational inertia (massive excitations) reproduces cold dark matter behavior, while a macroscopic order parameter, dynamically frozen by thermodynamic friction, generates accelerated cosmic expansion.

The block-diagonal tensor representation

$$\mathcal{G}_{MN} = g_{\mu\nu} \oplus f_{\mu\nu} \quad (127)$$

serves as a formal organizational tool to enforce orthogonality between matter and information, preventing trivial complex superpositions rather than acting as a literal 8D extension of macroscopic spacetime. Physical observables remain firmly associated with the physical sector.

The model is constructed to be testable. Viability requires consistency with structure formation, cosmic expansion history, gravitational-wave constraints, and the absence of observable fifth forces. These conditions restrict the allowed parameter space of the negentropic sector.

An interpretive layer was introduced in which the  $f_{\mu\nu}$  manifold is associated with maintained informational order. Distributed cryptographic consensus was discussed only as a localized thermodynamic analogy illustrating how irreversible physical dissipation can maintain persistent records in information-processing systems. This analogy does not enter the gravitational action and does not contribute to the cosmological stress-energy budget.

Several open problems remain. A full perturbative analysis is required to establish stability and determine the detailed evolution of density perturbations. In addition, a deeper connection to fundamental theories of quantum gravity or statistical mechanics is needed to derive the information sector from first principles.

Despite these limitations, the framework provides a controlled, falsifiable phenomenological setting in which dark matter, dark energy, and information-theoretic thermodynamics can be synthesized within a unified effective description.

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### Appendix A: Macroscopic Information, Emergy, and Bimetric Value

The block-diagonal tensor formulation used above admits a macroscopic interpretation closely related to a two-dimensional diagnostic vector in an orthogonal value space:

$$\vec{V}_{\text{val}} = \begin{pmatrix} v_{\text{phys}} \\ v_{\text{info}} \end{pmatrix}. \quad (A1)$$

The components  $v_{\text{phys}}$  and  $v_{\text{info}}$  are macroscopic diagnostic variables and should not be confused with the cosmological scale factors  $a(t)$  and  $b(t)$ .

The  $v_{\text{phys}}$  component, corresponding to the  $g_{\mu\nu}$  physical axis, represents embodied thermodynamic reality: matter, radiation, biological organization, geochemical structure, and the accumulated memory of solar work. The  $v_{\text{info}}$  component, corresponding to the  $f_{\mu\nu}$  axis, represents orthogonal informational order.

This appendix does not introduce additional fundamental dynamics. Instead, it provides an interpretive layer connecting the effective field theory to a broader macroscopic thermodynamic-information picture.

We introduce a coarse-grained emergy density  $\mathcal{E}_{\odot}$ , representing history-dependent solar work stored in physical and biological structures. A phenomenological definition is

$$\mathcal{E}_{\odot}(x, t) = \int_{-\infty}^t K(t, t') \Phi_{\odot}(x, t') dt', \quad (A2)$$

where  $\Phi_{\odot}$  is absorbed solar-energy flux and  $K(t, t')$  is a memory kernel encoding persistence, transformation efficiency, and ecological or geological storage.

The physical-axis value density may then be represented schematically as

$$v_{\text{phys}}(x, t) \sim \mathcal{E}_{\odot}(x, t). \quad (A3)$$

This notation avoids conflict with the cosmological scale factor  $a(t)$ .

This identification is not meant to replace stress-energy in general relativity. Rather, it provides a thermodynamic interpretation of the physical sector as a reservoir of embodied work.

At the effective level, one may write the macroscopic stress-energy tensor as:

$$T_{\mu\nu}^{\text{phys}} = T_{\mu\nu}^{\text{SM}} + T_{\mu\nu}^{\text{bio}} + T_{\mu\nu}^{\text{emergy}}, \quad (A4)$$

where  $T_{\mu\nu}^{\text{bio}}$  and  $T_{\mu\nu}^{\text{emergy}}$  are coarse-grained descriptions valid only at macroscopic ecological or planetary scales.

#### 1. Information Orthogonality and the Independent Axis

The metric  $f_{\mu\nu}$  is interpreted as the geometric carrier of orthogonal informational order. The block-diagonal

structure of the state tensor,

$$\mathcal{G}_{MN} = g_{\mu\nu} \oplus f_{\mu\nu}, \quad (\text{A5})$$

denotes a  $\pi/2$ -like rotation in an abstract value phase space. It ensures that informational order and physical mass-energy remain mathematically distinct, interacting dynamically but never kinematically.

Recalling our topological time derivative ( $\chi' = d\chi/d\theta$ ), we define a coarse-grained information-order density,

$$\mathcal{I} = \frac{1}{V_f} \int_{\Sigma_f} d^3x \sqrt{\gamma_f} \left[ \frac{1}{2} (\chi')^2 + U(\chi) \right], \quad (\text{A6})$$

where  $V_f$  is the informational-sector spatial volume,

$$V_f = \int_{\Sigma_f} d^3x \sqrt{\gamma_f}. \quad (\text{A7})$$

A phenomenological projection from real thermodynamic work into orthogonal information order may be written as

$$\mathcal{I} = \kappa_I \Pi_{\perp} [\mathcal{E}_{\odot}], \quad (\text{A8})$$

where  $\Pi_{\perp}$  is an abstract orthogonalization map and  $\kappa_I$  is an energy-information conversion coefficient.

This expression may be understood as a formal analogue of irreversible physical work being encoded into durable information. Cryptographic proof systems, including proof-of-work architectures, provide a terrestrial analogy: they transform dissipated physical work into an ordered, difficult-to-alter informational record. In the present cosmological model, however, such systems are not assumed to be the source of dark matter or dark energy. They serve only as an illustrative analogue of the more general principle of thermodynamic work projected into maintained information order.

## 2. Negative Entropy as the Source of Hidden Curvature

The maintenance of macroscopic order requires the continuous export of entropy, establishing the orthogonal informational sector as an open dissipative structure. Following the Brillouin-Schrödinger paradigm, information acts as negative entropy (negentropy). The global thermodynamic constraint across the bimetric manifold requires:

$$dS_{\text{tot}} = dS_g + dS_f \geq 0, \quad (\text{A9})$$

where  $S_g$  and  $S_f$  are the entropies associated with the physical and negentropic metrics, respectively.

For the informational sector to progressively crystallize state updates, its local entropy evolution must be strictly negative:

$$\dot{S}_f < 0. \quad (\text{A10})$$

This violation of local entropy maximization is dynamically permitted only because it is coupled to massive, compensating thermal dissipation in the visible metric:

$$\dot{S}_g \gg |\dot{S}_f|. \quad (\text{A11})$$

This fundamental asymmetry has profound geometric consequences. In standard relativity, positive entropy production is associated with the dispersion of mass-energy. Conversely, the accumulation of negative entropy ( $\dot{S}_f < 0$ ) within the informational metric implies an escalating local ordering.

We formally associate the scalar order field  $\chi$  with the negentropic density of the orthogonal manifold:

$$\chi \propto \exp\left(-\frac{S_f}{k_B}\right). \quad (\text{A12})$$

Consequently, the negentropic sector does not violate the second law of thermodynamics; rather, it inverts its dynamical consequence. The negative entropy of the  $f_{\mu\nu}$  metric acts as the fundamental source for the stress-energy tensor  $T_{\mu\nu}^{\mathcal{I}}$ , exerting a topological pressure that projects onto the visible universe as dark energy.

## 3. Orthogonal Value Functional

The macroscopic interpretive norm associated with the orthogonal value vector

$$\vec{V}_{\text{val}} = \begin{pmatrix} v_{\text{phys}} \\ v_{\text{info}} \end{pmatrix} \quad (\text{A13})$$

is its Euclidean magnitude,

$$|\vec{V}_{\text{val}}| = \sqrt{v_{\text{phys}}^2 + v_{\text{info}}^2}. \quad (\text{A14})$$

Here  $v_{\text{phys}}$  and  $v_{\text{info}}$  are macroscopic diagnostic quantities and should not be confused with the cosmological scale factors  $a(t)$  and  $b(t)$ .

In the present framework, the analogous real observable is an orthogonal value functional combining thermodynamic embodiment and informational order:

$$\mathcal{V}_{\text{HB}} = \int_{\Sigma_t} d^3x \sqrt{\gamma_g} [\mathcal{E}_{\odot}^2 + \alpha_I \mathcal{I}^2]^{1/2}. \quad (\text{A15})$$

Here  $\mathcal{I}$  denotes an information-order density and  $\alpha_I$  is assigned whatever mass dimension is required so that

$$[\alpha_I \mathcal{I}^2] = [\mathcal{E}_{\odot}^2]. \quad (\text{A16})$$

If  $\mathcal{E}_\odot$  is an energy density, then  $\mathcal{V}_{\text{HB}}$  has dimensions of energy. The functional is therefore a macroscopic diagnostic, not a dimensionless conserved charge or a new gravitational source.

The functional  $\mathcal{V}_{\text{HB}}$  is not a new fundamental interaction. It is a macroscopic diagnostic of joint physical-informational value. It formalizes the idea that a stable civilization, biosphere, or complex system cannot maximize informational abstraction while destroying its real thermodynamic base.

One may express a sustainability condition as

$$\frac{d\mathcal{V}_{\text{HB}}}{dt} > 0, \quad \frac{d}{dt} \int_{\Sigma_t} d^3x \sqrt{\gamma_g} \mathcal{E}_\odot \geq 0. \quad (\text{A17})$$

The first condition states that total bimetric value grows. The second states that such growth must not be achieved by depleting the physical-axis energy base. This provides a formal place for the philosophical concept of *Homo Biodiversitas*: an observer or civilization that increases informational order while preserving or regenerating the thermodynamic and ecological substrate from which that order is derived.

## Appendix B: Thermodynamic Dissipation as a Macroscopic Analogy

This appendix is interpretive. It provides a macroscopic analogy for irreversible work producing persistent information, but it does not enter the gravitational action in Eq. (6), does not contribute appreciably to the cosmological stress-energy budget, and is not assumed to source dark matter or dark energy.

The orthogonal informational sector introduced above acts as an effective geometric description of maintained macroscopic order. To ground this concept phenomenologically, it is instructive to examine systems where irreversible physical work is systematically converted into persistent, orthogonal informational structures. Distributed cryptographic consensus mechanisms provide a rigorous, albeit localized, thermodynamic analogue.

We stress that Proof-of-Work plays no dynamical role in the cosmological model. Its energy scale is negligible in comparison to cosmological densities, and no direct contribution to gravitational dynamics is implied. Its role is purely illustrative: it provides a controlled example of the mapping

$$\text{irreversible work} \longrightarrow \text{persistent information}. \quad (\text{B1})$$

Let  $\mathcal{P}_{\text{diss}}(t)$  denote the total exergy dissipation rate of a localized, fault-tolerant information network. The cumulative dissipated physical energy is

$$E_{\text{diss}}(t) = \int_{t_0}^t \mathcal{P}_{\text{diss}}(t') dt'. \quad (\text{B2})$$

The accumulated work securing the ledger can be represented by a monotonic quantity  $W_{\text{cum}}(t)$ , proportional to the integrated difficulty-weighted computational effort. A dimensionless measure of persistent information order is then

$$\mathcal{I}_{\text{PoW}}(t) = \log \left( 1 + \frac{W_{\text{cum}}(t)}{W_0} \right), \quad (\text{B3})$$

where  $W_0$  is a reference scale.

The logarithmic form reflects the fact that security is governed by relative cost: each additional unit of accumulated work increases the difficulty of rewriting the ledger by a multiplicative factor.

The rate of order accumulation is

$$\dot{\mathcal{I}}_{\text{PoW}} = \frac{\dot{W}_{\text{cum}}}{W_0 + W_{\text{cum}}}. \quad (\text{B4})$$

This construction highlights a key feature: the system generates an effective arrow of time. Past states become increasingly resistant to modification as cumulative work grows.

An effective thermodynamic cost of persistent memory may be defined as

$$C_{\text{mem}} = \frac{E_{\text{diss}}}{\mathcal{N}_{\text{state}}}, \quad (\text{B5})$$

where  $\mathcal{N}_{\text{state}}$  denotes the number of cryptographically sealed macroscopic state transitions within the topological register.

The analogy with the orthogonal-value construction introduced earlier can be expressed through a localized functional:

$$\mathcal{V}_{\text{cons}} = [\mathcal{E}_\odot^2 + \alpha_I \mathcal{I}_{\text{cons}}^2]^{1/2}, \quad (\text{B6})$$

where  $\mathcal{I}_{\text{cons}}$  isolates the informational entropy reduction achieved through distributed consensus.

This expression mirrors the general vector norm structure

$$|\vec{V}| = \sqrt{a^2 + b^2}, \quad (\text{B7})$$

with thermodynamic embodiment and informational order contributing as orthogonal components. Fundamentally, physical energy does not strictly encode information in its own native units. Instead, we invoke a macroscopic analogue to Landauer's Principle [24]. While Landauer established the minimum thermal dissipation required to erase a bit, our bimetric interaction operates as the inverse generative process: the continuous, irreversible dissipation of exergy in  $g_{\mu\nu}$  is the required physical cost to crystallize and seal topological information within  $f_{\mu\nu}$  [28]. Recent phenomenological frameworks, such as the D-Trinity protocol combining cryptographic hashing (SHA-256) with decentralized relays, provide a topological blueprint for how such rigid informational ledgers achieve macroscopic thermodynamic stability and phase transitions [29].

In this sense, Proof-of-Work consensus mechanisms provide a localized laboratory-scale example of the general principle underlying the orthogonal informational sector: persistent informational order requires a physical substrate, an energetic cost, and an irreversibility mechanism. This analogy is strictly operational and does not imply any direct cosmological role.

### 1. Phenomenological Friction and the Bimetric Landauer Limit

In standard information theory, Landauer's principle defines the minimum thermal cost of erasing a bit [24]. We postulate an inverted, macroscopic analogue: the crystallization of topological order requires a continuous, irreversible expenditure of physical exergy.

Let  $\delta W_{\text{diss}}$  be the irreversible physical work dissipated in the visible metric  $g_{\mu\nu}$ , and  $\delta\chi$  be the corresponding increment of negentropic order in  $f_{\mu\nu}$ . We hypothesize a fundamental threshold relation:

$$\delta W_{\text{diss}} = \mathcal{T}^* \delta\chi, \quad (\text{B8})$$

where  $\mathcal{T}^*$  acts as an effective topological tension (with dimensions of energy per unit order), representing the critical thermodynamic cost required to rigidly anchor an informational state against natural entropic decay.

In the language of field theory, the spontaneous evolution of the order parameter  $\chi$  is governed by the Euler-Lagrange equation. To maintain the system away from thermal equilibrium (preventing the decay of  $\chi$  down its potential  $U$ ), the physical dissipation must act as an effective restorative force. We capture this by introducing a dissipative current  $J_{\text{diss}}$  into the scalar equation of motion:

$$\square_f \chi - \frac{dU}{d\chi} + \frac{1}{\sqrt{-f}} \frac{\delta S_{\text{mix}}}{\delta\chi} = J_{\text{diss}}. \quad (\text{B9})$$

We model this current covariantly as being strictly sourced by the entropy production rate  $\Phi_g$  within the physical metric through the portal interaction  $S_{\text{portal}}$ . Taking  $n^\lambda$  as the unit 4-velocity vector of the macroscopic informational flow, the friction current is:

$$J_{\text{portal}} = -\gamma(\Phi_g) n^\lambda \nabla_\lambda \chi, \quad (\text{B10})$$

where  $\gamma(\Phi_g)$  is the bimetric friction coefficient, strictly monotonically increasing with the physical power dissipated in  $g_{\mu\nu}$ . The back-reaction of this term onto  $T_{\mu\nu}^{\text{SM}}$  is precisely what accounts for the macroscopic loss of available exergy in the physical universe when maintaining topological records.

Crucially, in the rest frame of the homogeneous FLRW background of the  $f$ -metric ( $ds_f^2 = -N_f^2 dt^2 + b^2 d\vec{x}^2$ ), the unit normal vector is  $n^\lambda = (1/N_f, 0, 0, 0)$ . Therefore, the directional derivative is exactly:

$$n^\lambda \nabla_\lambda \chi = \frac{1}{N_f} \dot{\chi} \equiv \chi'. \quad (\text{B11})$$

This shows that the friction current is naturally expressed in terms of the  $f$ -proper-time derivative rather than the visible-sector cosmic-time derivative. Any additional interpretation of this relative clock rate as a discrete informational update parameter is phenomenological and does not modify the covariant definition of  $\chi'$ . Substituting this along with the Hassan–Rosen interaction derivative yields the complete effective friction equation:

$$\square_f \chi + \gamma(\Phi_g) \chi' - \frac{dU}{d\chi} - \frac{\sqrt{-g}}{\sqrt{-f}} \frac{\partial V_{\text{int}}}{\partial\chi} = 0, \quad (\text{B12})$$

where the volume determinant ratio  $\sqrt{-g}/\sqrt{-f}$  (which evaluates to  $(N_f r^3)^{-1}$  in the cosmological background) rigorously accounts for the projection of the interaction potential from the physical to the informational manifold, and

$$V_{\text{int}} = M_{\text{eff}}^2 m^2(\chi) \sum_{n=0}^4 \beta_n e_n(S), \quad (\text{B13})$$

and therefore, for  $m^2(\chi) = m_0^2 \exp(-\beta_\chi \chi / M_{\text{Pl}})$ ,

$$\frac{\partial V_{\text{int}}}{\partial\chi} = -\frac{\beta_\chi}{M_{\text{Pl}}} M_{\text{eff}}^2 m_0^2 \exp\left(-\frac{\beta_\chi \chi}{M_{\text{Pl}}}\right) \sum_{n=0}^4 \beta_n e_n(S). \quad (\text{B14})$$

Each term in Eq. (B12) has mass dimension  $M^3$ , provided  $[\gamma] = M$ .

This equation dictates the macroscopic phase transition of the negentropic sector. If the physical dissipation  $\Phi_g$  is below a critical threshold, the friction term  $\gamma(\Phi_g) \chi'$  is insufficient to overcome the natural potential gradient  $dU/d\chi$ , leaving the informational state fluid and non-consensual. However, when  $\gamma(\Phi_g)$  is large compared with the characteristic inverse relaxation time of the potential, the motion of  $\chi$  is overdamped:

$$|\chi''| \ll \gamma |\chi'|, \quad \frac{\chi'^2}{2} \ll U(\chi). \quad (\text{B15})$$

In this regime the scalar behaves approximately as a vacuum component,

$$w_\chi = \frac{\chi'^2/2 - U(\chi)}{\chi'^2/2 + U(\chi)} \simeq -1, \quad (\text{B16})$$

while simultaneously modulating the bimetric interaction scale through  $m^2(\chi)$ .

### Appendix C: Quantum Decoherence as Micro-Bimetric Interaction

As a speculative interpretation, one may ask whether the same bimetric language can provide intuition for quantum measurement or decoherence. This possibility is not part of the cosmological model developed in the main text. No modified Schrödinger equation, Lindblad equation, or microscopic collapse model is derived here. In standard

quantum theory, the state of a system prior to measurement is described by a complex probability amplitude,  $\psi(x, t) = \psi_R + i\psi_I$ . Within our geometric correspondence, where the state tensor separates into orthogonal physical and informational metrics ( $\mathcal{G}_{MN} = g_{\mu\nu} \oplus f_{\mu\nu}$ ), this complex nature is no longer a purely statistical artifact, but a reflection of the underlying bimetric topology.

We postulate that quantum superposition represents a regime where an excitation propagates predominantly within the informational metric  $f_{\mu\nu}$ , unconstrained by the macroscopic thermodynamic dissipation of the physical axis. In this state, the particle explores the non-local topological graph of the informational sector, providing a geometric basis for quantum entanglement. Particles separated by vast spatial distances in  $g_{\mu\nu}$  may remain topologically adjacent in  $f_{\mu\nu}$ .

The act of quantum measurement (decoherence) re-

quires a macroscopic apparatus, inevitably involving irreversible thermodynamic dissipation in the visible metric. Thus, wavefunction collapse is interpreted as a micro-scale phase transition induced by the bimetric mixing potential  $S_{\text{mix}}$ . The thermal friction of the measurement forces the delocalized informational state in  $f_{\mu\nu}$  to precipitate onto the physical axis  $g_{\mu\nu}$ . Decoherence is, therefore, the microscopic inverse of cryptographic consensus: thermal dissipation forcing a projection from the informational axis ( $f_{\mu\nu}$ ) down to the physical axis ( $g_{\mu\nu}$ ). This geometric interpretation is speculative and is not used in deriving the cosmological background or perturbation equations. Establishing any connection with quantum measurement would require an explicit microscopic open-system model, such as a derived Lindblad equation or a modified Schrödinger dynamics.

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